Karlsruher Institut für Technologie

Hybrid Quantum-Classical Optimization with the ProvideQ Toolbox

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Speaker Portfolio

Domenik Eichhorn
PhD Student at the Chair for
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Research Topics:
Quantum Software and Hybrid Algorithms

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Motivation – Quantum Supremancy

- **In theory,** quantum computers can utilize *quantum mechanical effects* to compute specific tasks faster than classical computers.

Use Cases:

- Cryptography
- Material Simulation
- Combinatorial Optimization

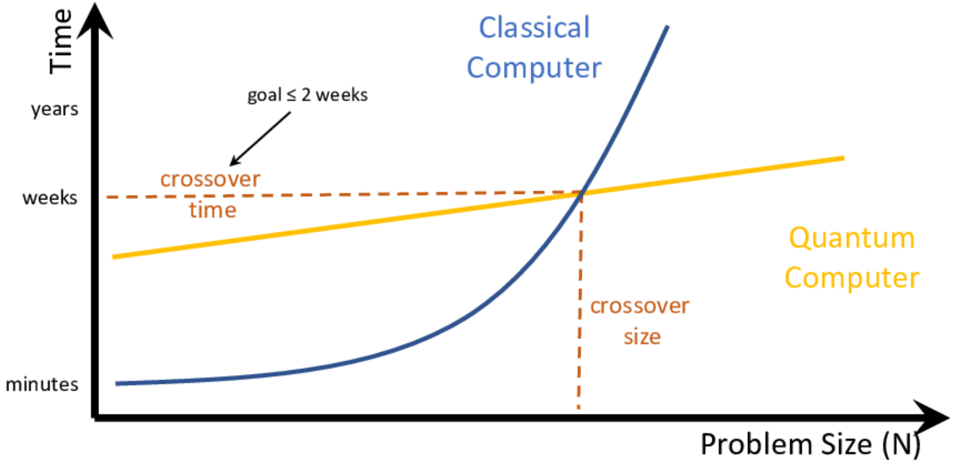
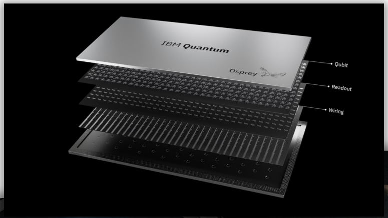


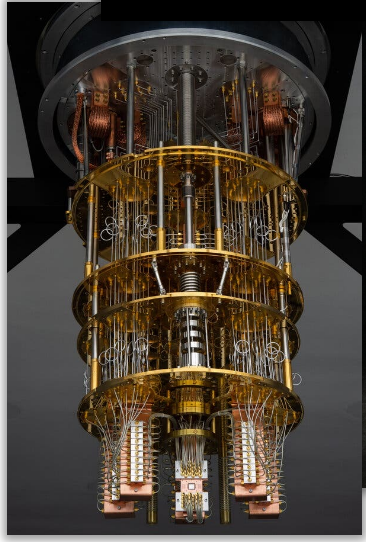
Image Src: T. Hoefler et al., Disentangling Hype from Practicality: On Realistically Achieving Quantum Advantage

Background – NISQ Quantum Hardware

Noisy
Intermediate
Scale
Quantum
(era)

- Noise – a lot …
- Scale – very small …
- Quantum – sometimes …






Background – Quantum Algorithms

„Superpolynomial Speedup“:

- Shor → Prime Factorization
- Deutsch-Josza → Check if Function is Constant

„Quadratic Speedup“:

- Grover → Search in unstructured Database

„Whats a speedup?“:

- QAOA → Approximation Algorithm
- VQE → Approximation Algorithm

Algorithms & Advantages in Combinatorial Optimizaiton

~~„Superpolynomial Speedup“:~~

- ~~Shor~~ → ~~Prime Factorization (Cryptography)~~
- ~~Deutsch-Josza~~ → ~~Check if Function is Constant (Useless)~~

„Quadratic Speedup“:

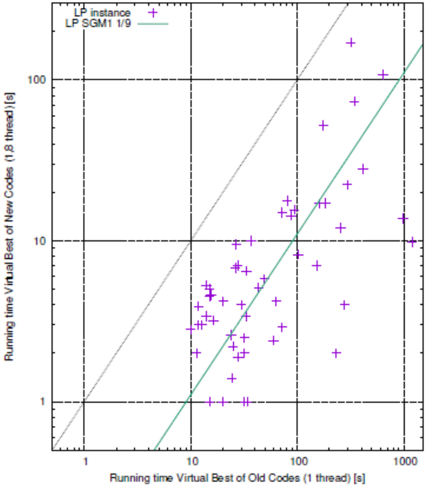
- Grover → Search in unstructured Database

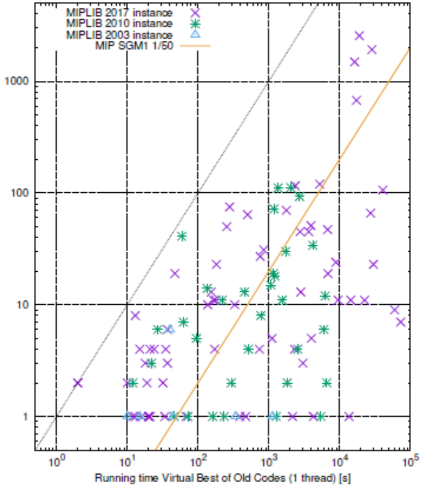
„Whats a speedup?“:

- QAOA → Approximation Algorithm
- VQE → Approximation Algorithm

What do we compete against?

- Classical Solvers have improved drastically in the recent years:





Source: Koch, T., Berthold, T., Pedersen, J., Vanaret, C.: Progress in mathematical programming solvers from 2001 to 2020. EURO Journal on Computational Optimization 10, 100031 (2022)

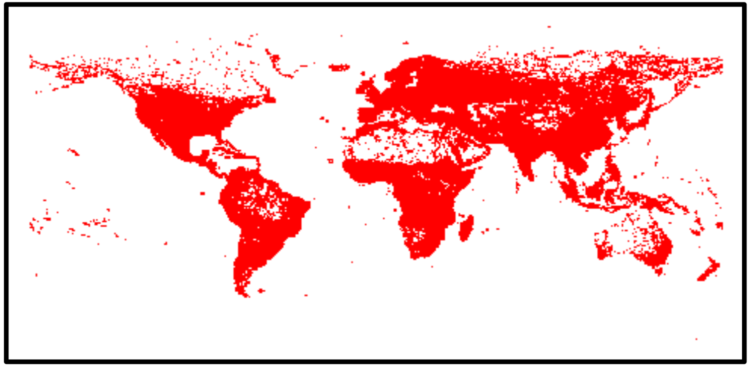
Example:
Speedup for (Mixed-Integer) Linear Programming Problems:

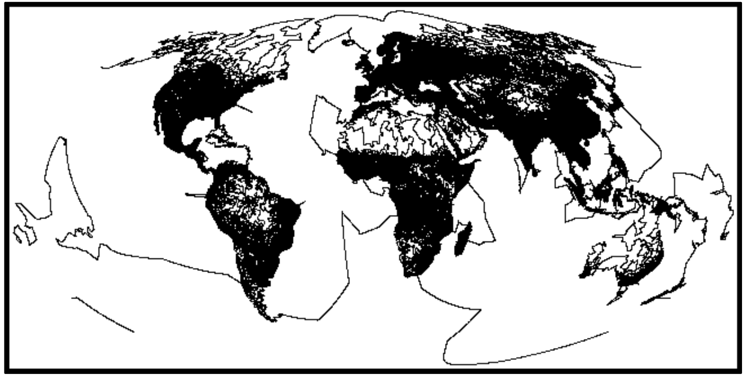
- LP: 9x Speedup
- MIP: 50x Speedup
- Hardware: 20x Speedup
- Combined LP: 180x
- Combined MIP: 1000x

SotA: Traveling Sales Person

Example – World TSP:

- 1,907,711 cities
- Lower Bound: 7,512,218,268
- Current best Solution: 7,515,755,956 (only 0.0471% greater than LB)





Source:
https://www.math.uwaterloo.ca/tsp/world/

Low Autocorrelation Binary Sequences (LABS)

- Also an Optimization problems, however, it „breaks“ much earlier.
- Even problems with only 60 variables are very hard to solve.

Consider a sequence $S = (s_1, \ldots, s_N)$ with $s_i = \pm 1$. The autocorrelations of S are defined as

$$C_k(S) = \sum_{i=1}^{N-k} s_i s_{i+k} \tag{1}$$

for $k = 0, 1, \ldots, N-1$, and the "energy" of S is defined as the sum of the squares of all off-peak correlations,

$$E(S) = \sum_{k=1}^{N-1} C_k^2(S)\,. \tag{2}$$

The *low-autocorrelation binary sequence* (LABS) problem is to find a sequence S of given length N that minimizes $E(S)$ or, equivalently, maximizes the *merit factor*

$$F(S) = \frac{N^2}{2E(S)}\,. \tag{3}$$

Source: Tom Packebusch & Stephan Mertens
Low Autocorrelation Binary Sequences
https://arxiv.org/abs/1512.02475

PHYSICS

Evidence of scaling advantage for the quantum approximate optimization algorithm on a classically intractable problem

Ruslan Shaydulin[1]*, Changhao Li[1], Shouvanik Chakrabarti[1], Matthew DeCross[2], Dylan Herman[1], Niraj Kumar[1], Jeffrey Larson[3], Danylo Lykov[1,4], Pierre Minssen[1], Yue Sun[1], Yuri Alexeev[4], Joan M. Dreiling[2], John P. Gaebler[2], Thomas M. Gatterman[2], Justin A. Gerber[2], Kevin Gilmore[2], Dan Gresh[2], Nathan Hewitt[2], Chandler V. Horst[2], Shaohan Hu[1], Jacob Johansen[2], Mitchell Matheny[2], Tanner Mengle[2], Michael Mills[2], Steven A. Moses[2], Brian Neyenhuis[2], Peter Siegfried[2], Romina Yalovetzky[1], Marco Pistoia[1]

The quantum approximate optimization algorithm (QAOA) is a leading candidate algorithm for solving optimization problems on quantum computers. However, the potential of QAOA to tackle classically intractable problems remains unclear. Here, we perform an extensive numerical investigation of QAOA on the low autocorrelation binary sequences (LABS) problem, which is classically intractable even for moderately sized instances. We perform noiseless simulations with up to 40 qubits and observe that the runtime of QAOA with fixed parameters scales better than branch-and-bound solvers, which are the state-of-the-art exact solvers for LABS. The combination of QAOA with quantum minimum finding gives the best empirical scaling of any algorithm for the LABS problem. We demonstrate experimental progress in executing QAOA for the LABS problem using an algorithm-specific error detection scheme on Quantinuum trapped-ion processors. Our results provide evidence for the utility of QAOA as an algorithmic component that enables quantum speedups.

More Background – Implementing Quantum Algorithms

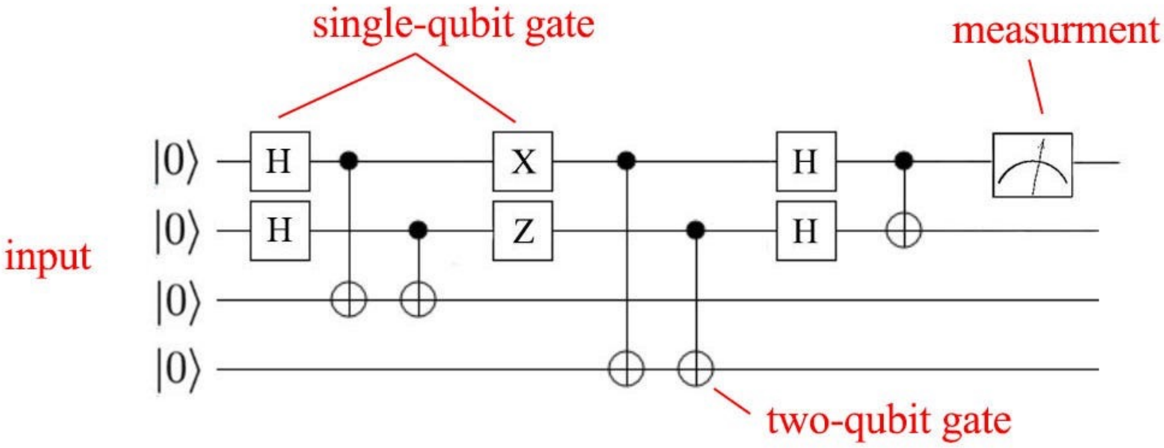


Image Src: https://jonathan-hui.medium.com/qc-programming-with-quantum-gates-8996b667d256

Hybrid Algorithms – A QAOA Example

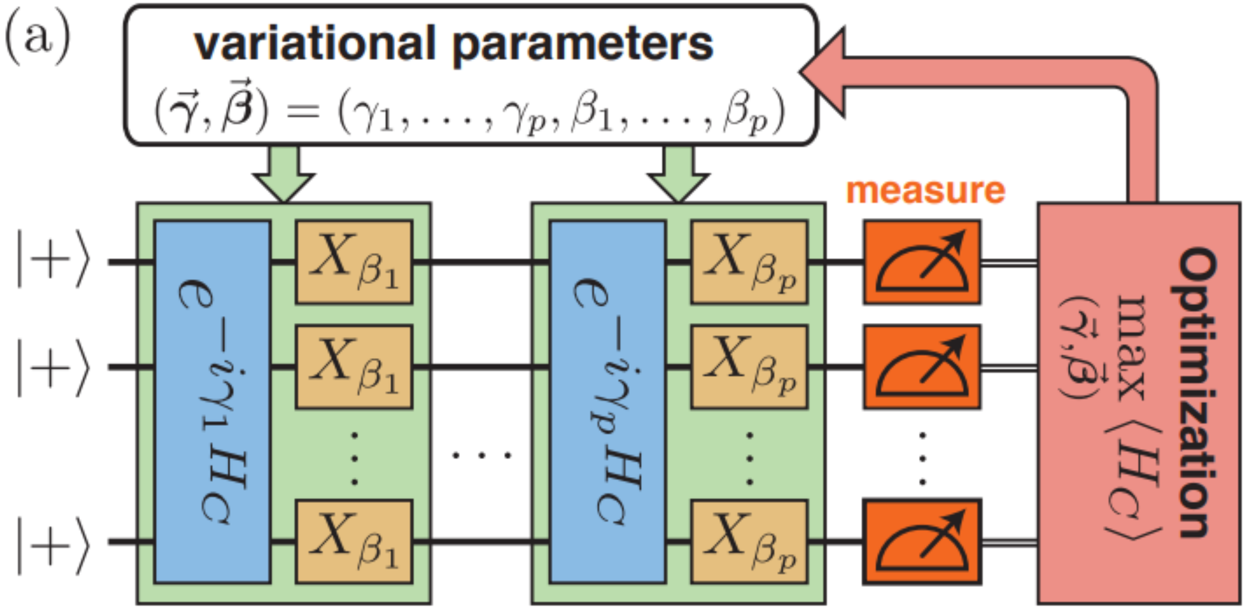
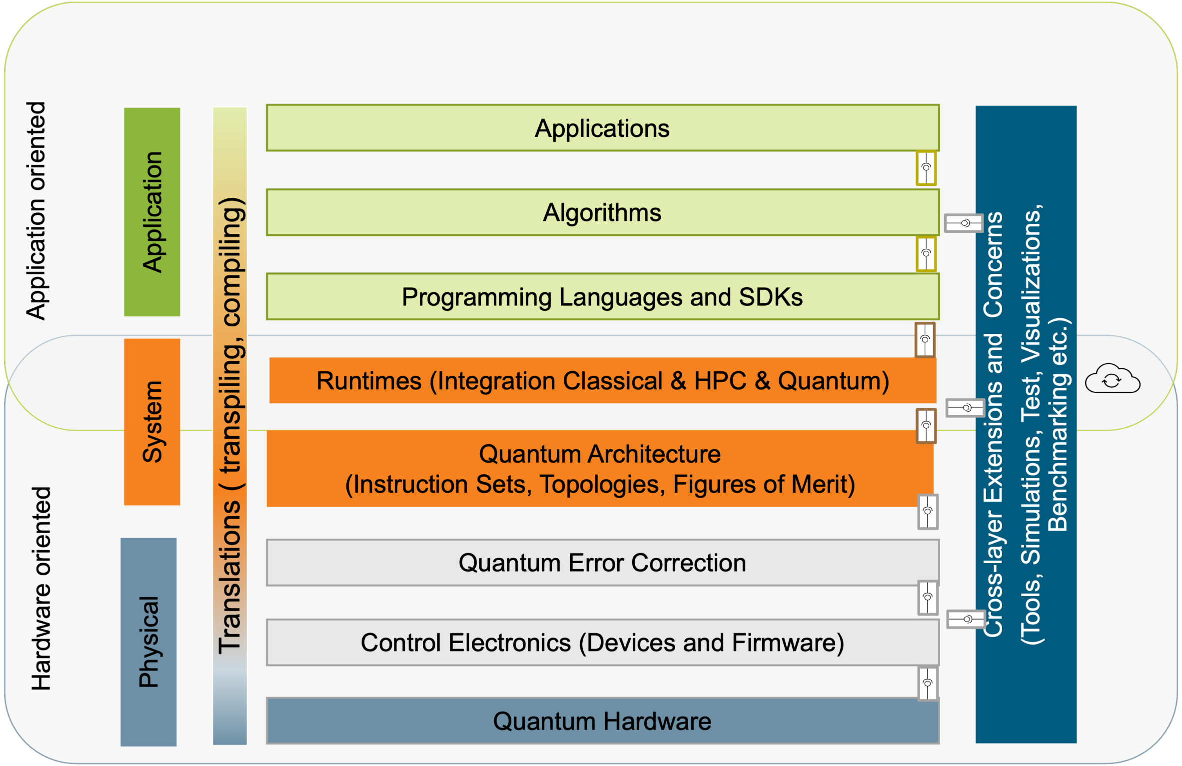


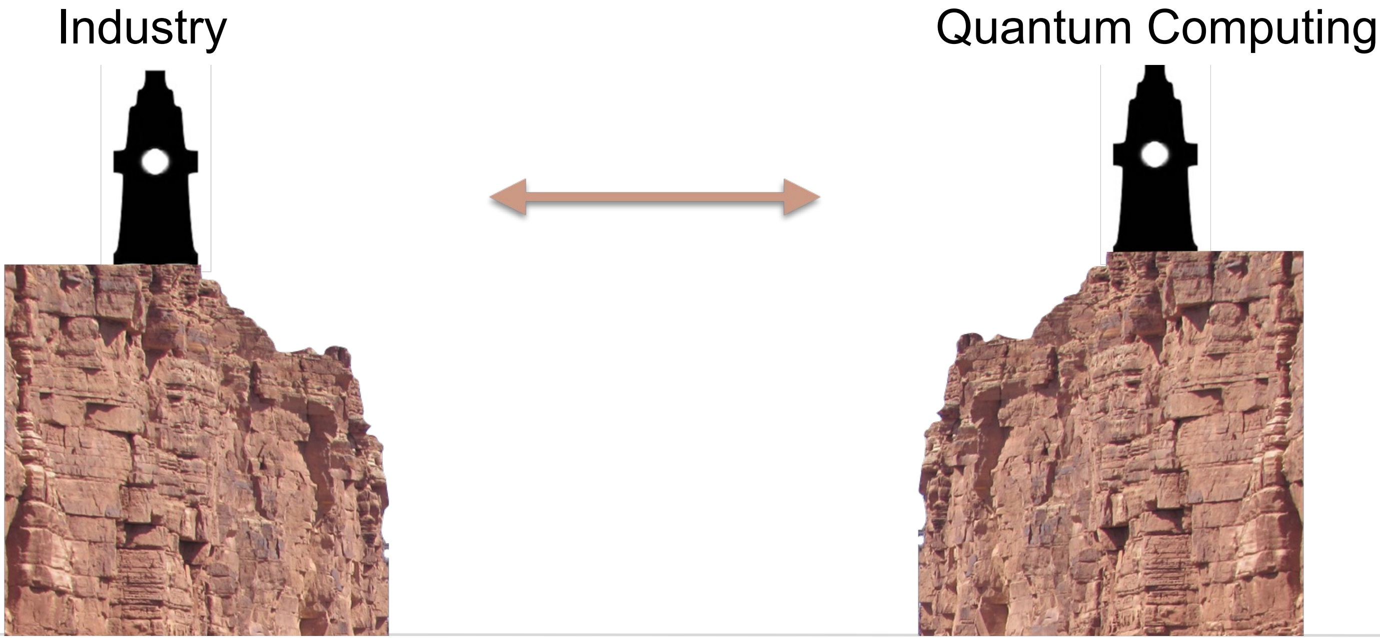
Image Src: https://medium.com/@chs.li.work/qaoa-quantum-approximate-optimization-algorithm-1cf6dabdd581

The „Quantum Software Stack“



Our Goal: „Enable the Enablers“

- Quantum Computing is very complicated…



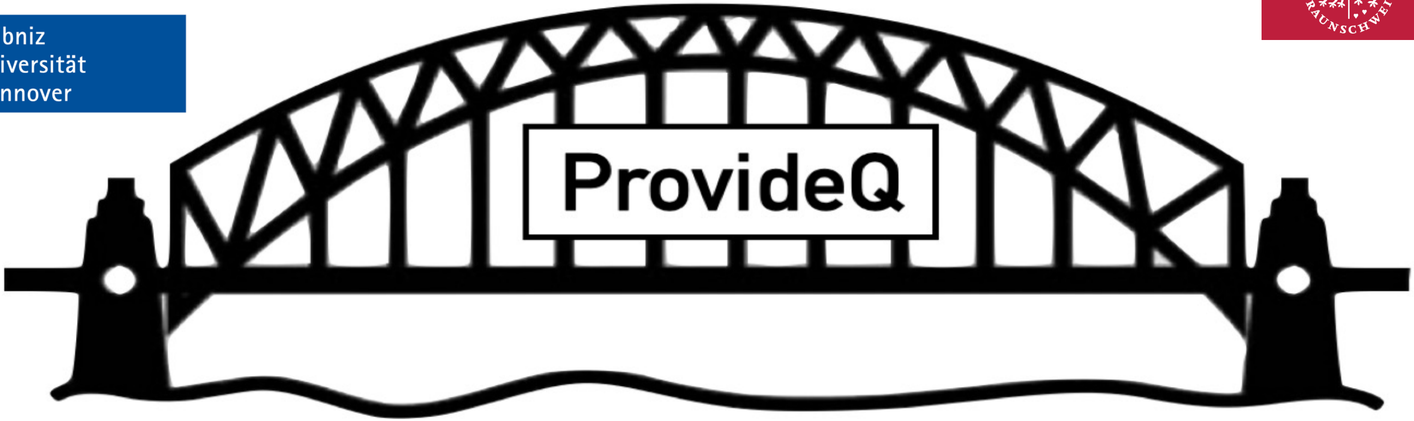
Our Goal: „Enable the Enablers“

Universität zu Köln

KIT Karlsruher Institut für Technologie

Technische Universität Braunschweig

Leibniz Universität Hannover



GAMS

4FLOW

Classical Optimization Frameworks

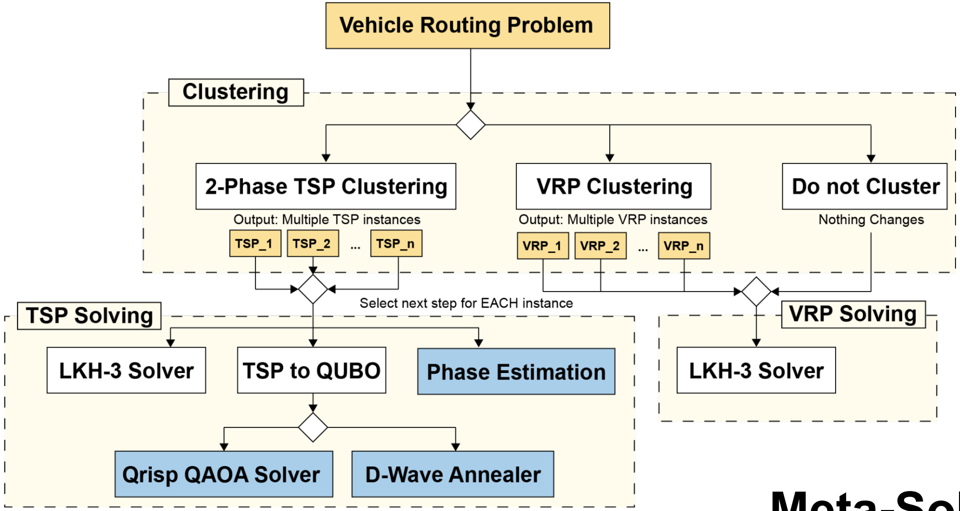
Optimization Framework

GUROBI OPTIMIZATION

Large Mathematical Model → Google OR-Tools → **Solution**

IBM CPLEX

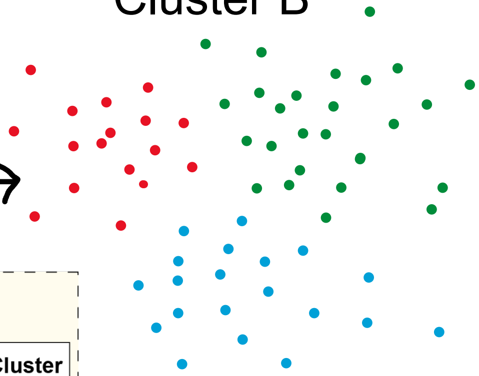
How to add Quantum? → Decompose Problems!

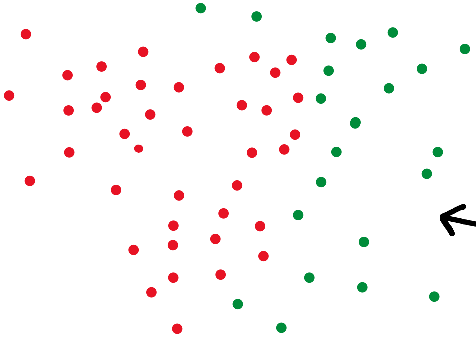


„Meta-Solving“

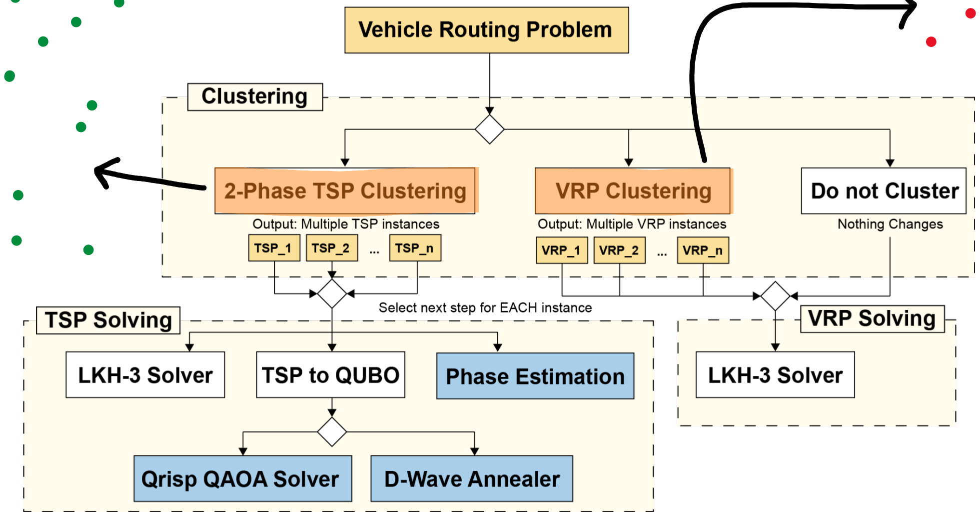
Meta Solving – Parallel Execution

Cluster B



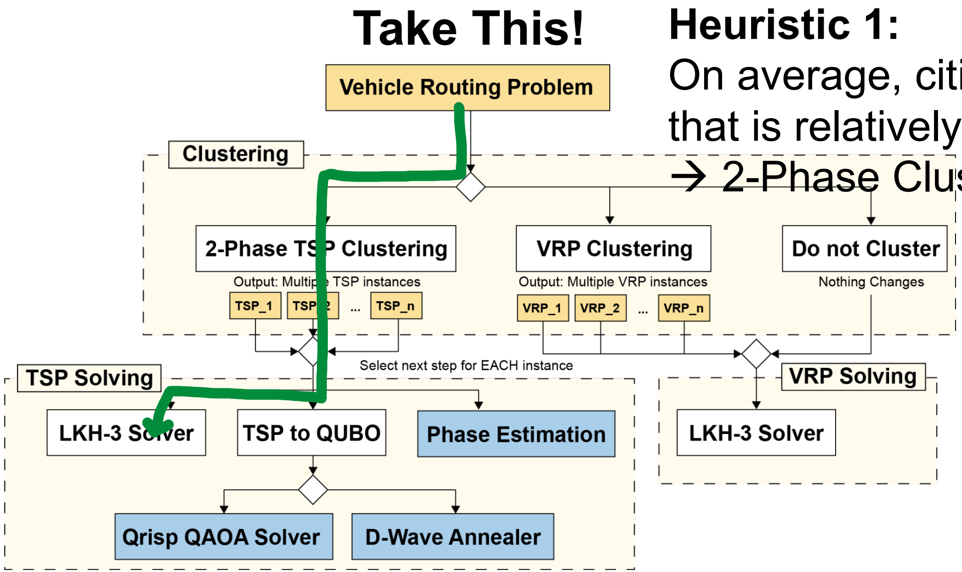


Cluster A



If user is unsure about the next step,
they can try out multiple solution at once.

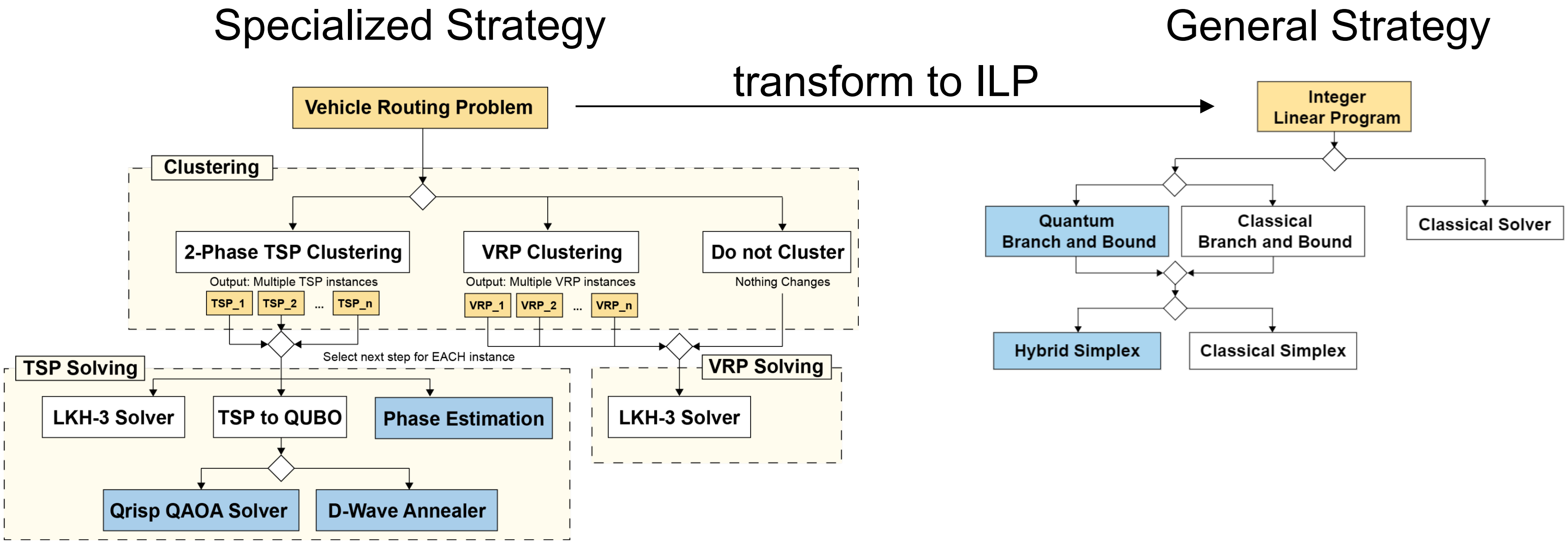
Meta Solving – Solution Path Suggestion



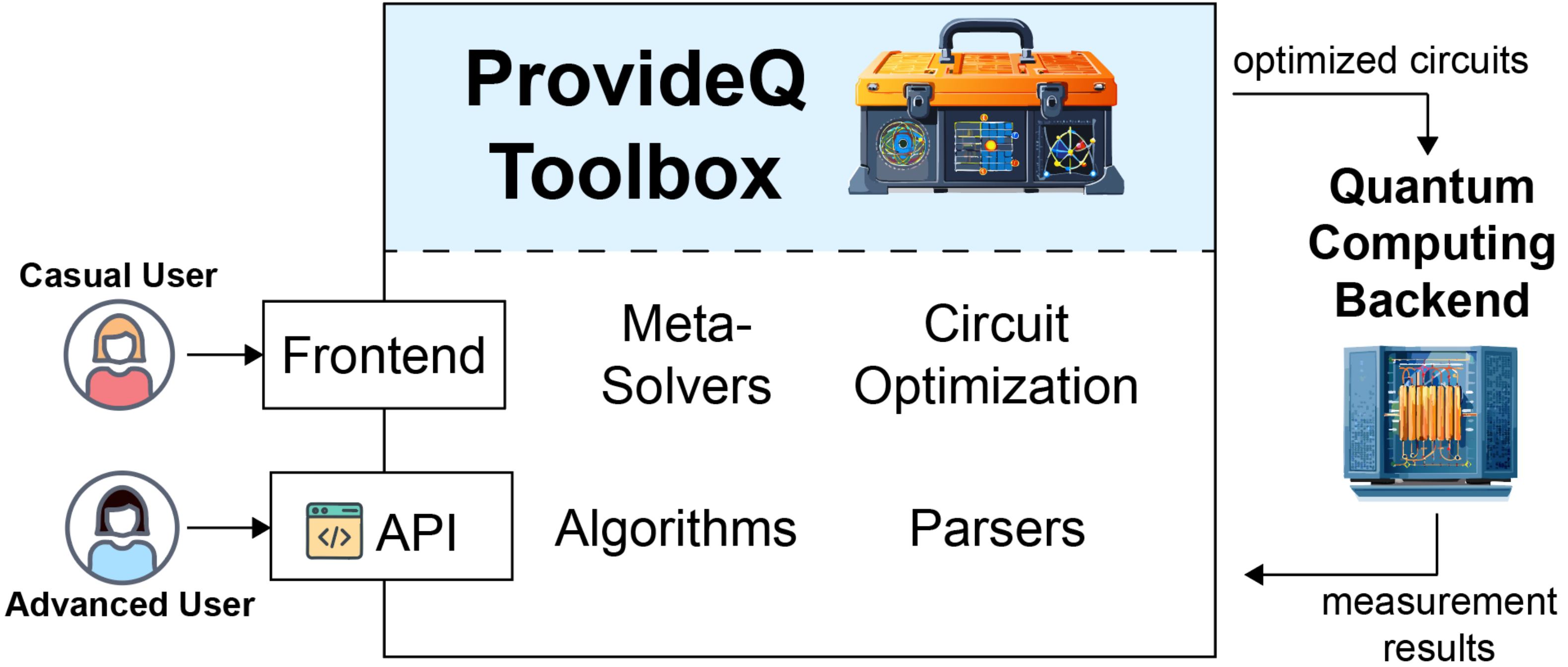
Heuristic 1:
On average, cities have low demand, that is relatively evenly split.
→ 2-Phase Clustering Approach is good.

Heuristic 2:
Problem too large for a Quantum Computer
→ Use a classical solver.

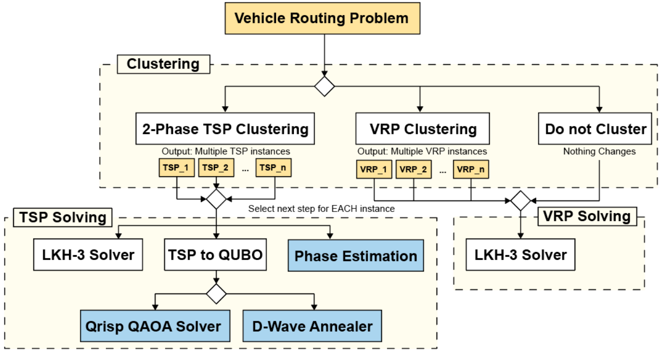
Meta Solving – Capsuling Strategies

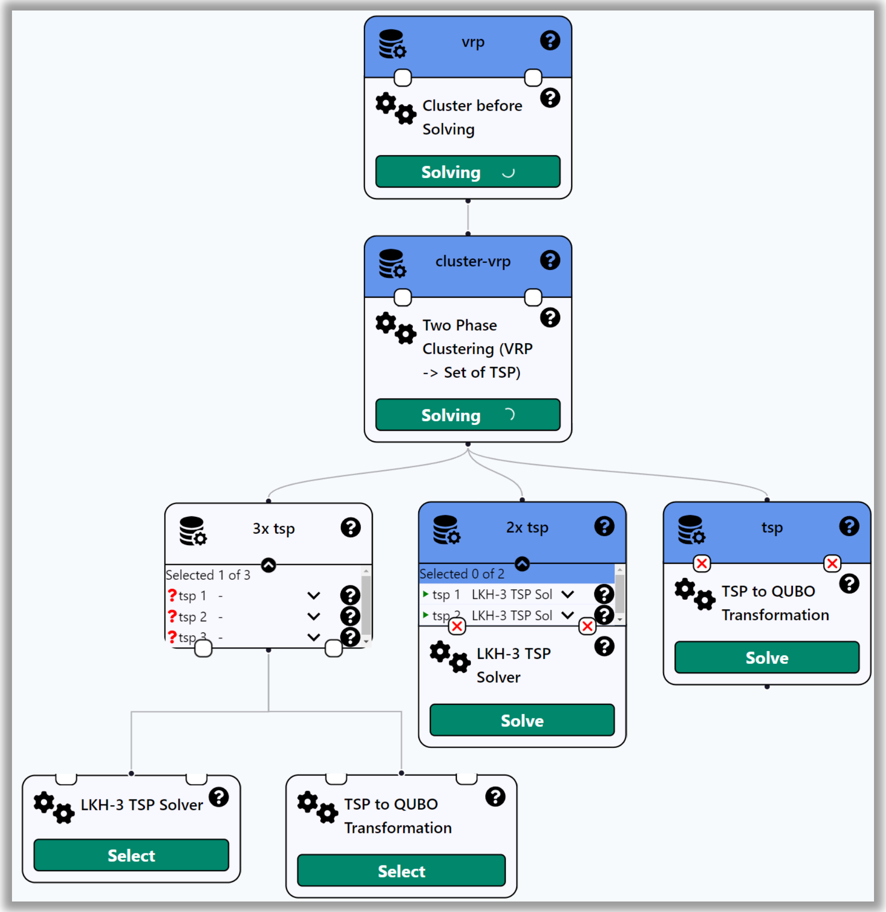


How to implement this?

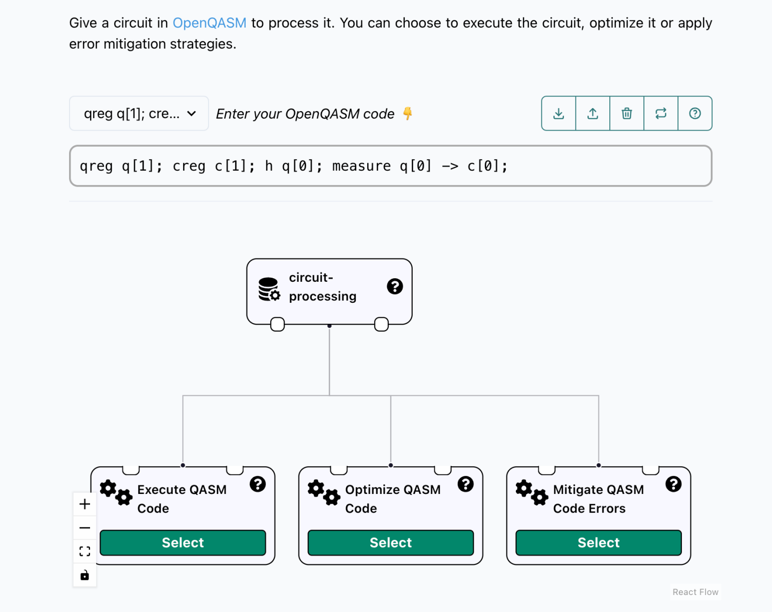


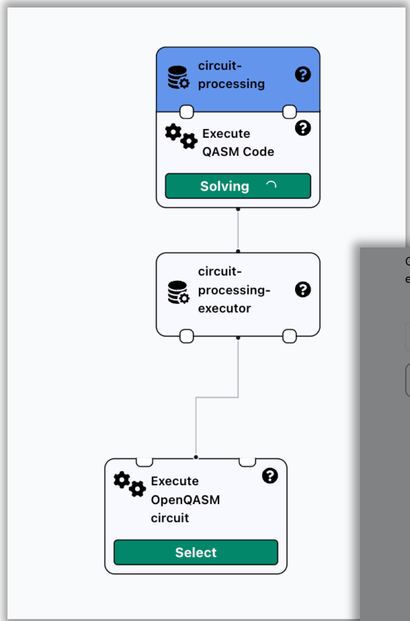
The ProvideQ Toolbox

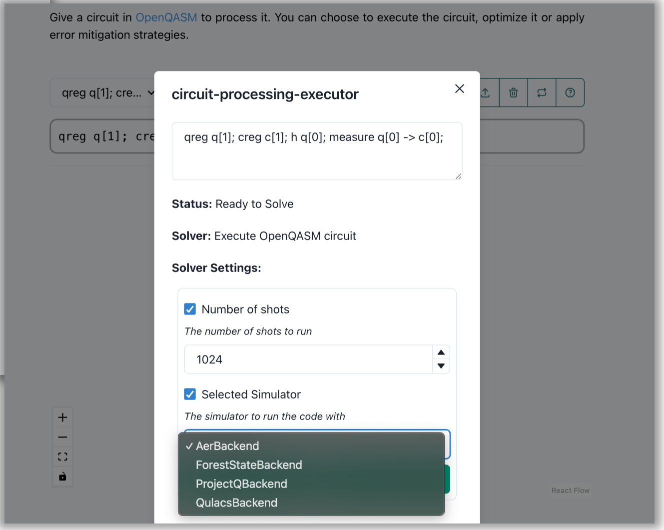




WIP: Quantum Circuit Execution & Optimization

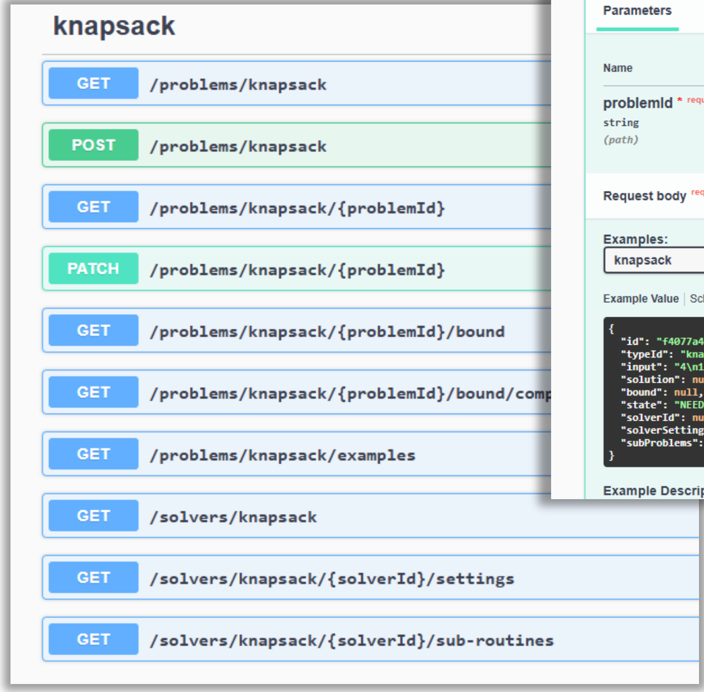


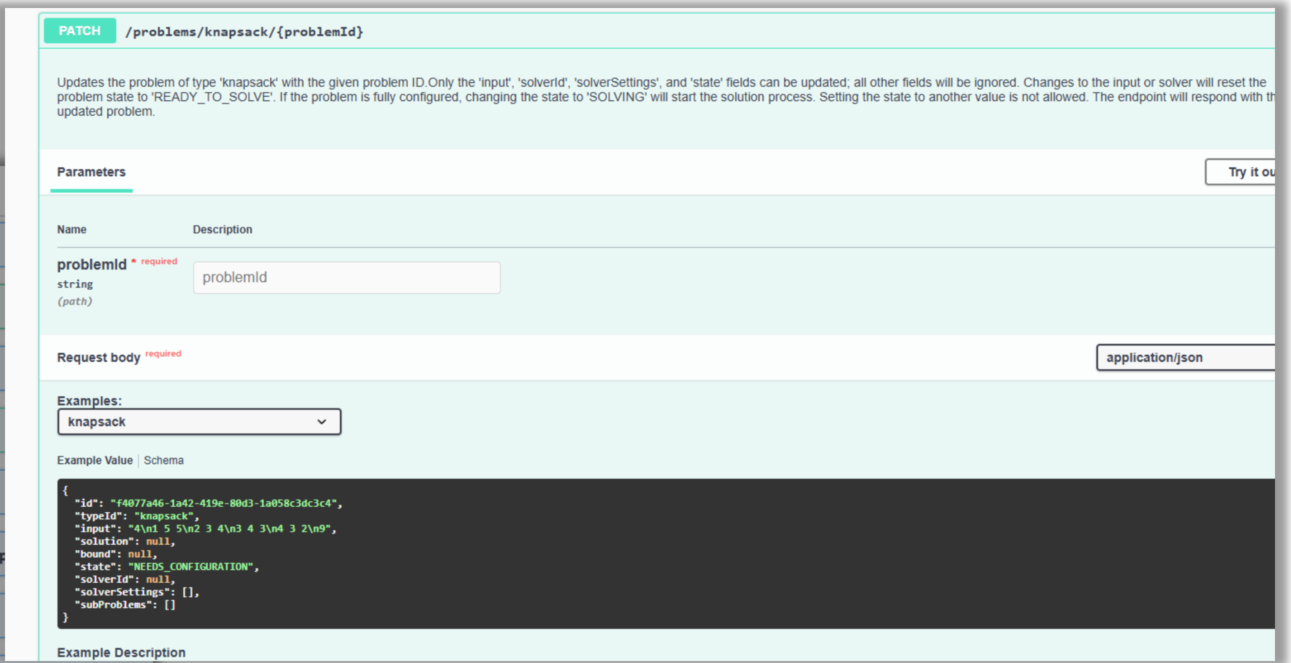




Try out the API:

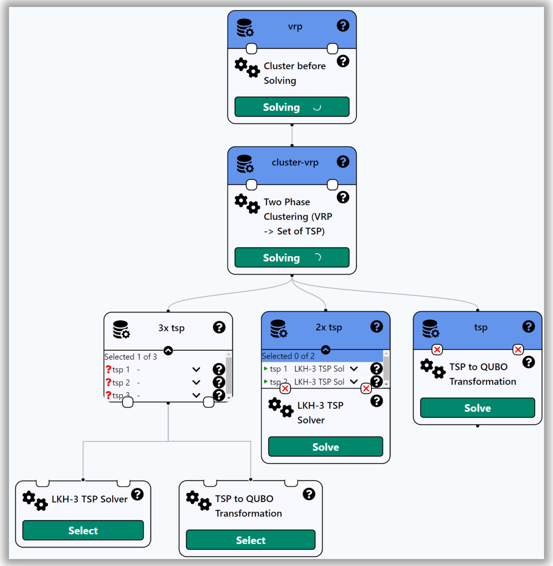
ProvideQ API





Conclusion

- Quantum Computing is not the answer to everything, but it has potential to provide supremacy for solving combinatorial optimizaiton problems.
- Meta-Solving and the ProvideQ toolbox provide prototypical concepts and implementations that allow an easy adaption for hybrid optimization.
- We have a hardware bottleneck.



Find us at:
https://github.com/ProvideQ
https://provideq.kit.edu/